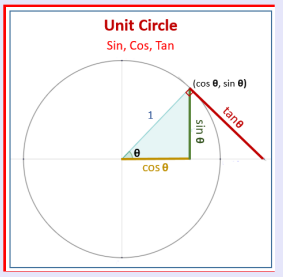
Trigonometry

Lecture 36



Feb 19-8:47 AM

Solve $4\sin^2\theta - 1 = 0$

$4\sin^2\theta = 1$

$\sin^2\theta = \frac{1}{4}$

$\sin\theta = \pm\sqrt{\frac{1}{4}}$

$\sin\theta = \pm\frac{1}{2}$

Ref. Angle 30°

$30^\circ + K\cdot 360^\circ$

$150^\circ + K\cdot 360^\circ$

$210^\circ + K\cdot 360^\circ$

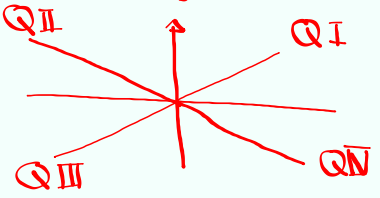
$330^\circ + K\cdot 360^\circ$

QI $\theta = 30^\circ + K\cdot 360^\circ$

QII $\theta = 180^\circ - 30^\circ + K\cdot 360^\circ$

QIII $\theta = 180^\circ + 30^\circ + K\cdot 360^\circ$

QIV $\theta = 360^\circ - 30^\circ + K\cdot 360^\circ$



$K = 0$ $\theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ$

$[0^\circ, 360^\circ)$

Nov 4-10:31 AM

Solve $4\cos^2 2\theta - 3 = 0$

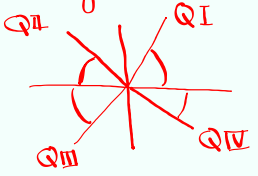
$4\cos^2 2\theta = 3$

$\cos^2 2\theta = \frac{3}{4}$

$\cos 2\theta = \pm\sqrt{\frac{3}{4}}$

$\cos 2\theta = \pm\frac{\sqrt{3}}{2}$

Ref. Angle 60°



QI $2\theta = 60^\circ + K\cdot 360^\circ$

$\theta = 30^\circ + K\cdot 180^\circ$

QII $2\theta = 180^\circ - 60^\circ + K\cdot 360^\circ$

$\theta = 60^\circ + K\cdot 180^\circ$

QIII $2\theta = 180^\circ + 60^\circ + K\cdot 360^\circ$

$\theta = 120^\circ + K\cdot 180^\circ$

QIV $2\theta = 360^\circ - 60^\circ + K\cdot 360^\circ$

$\theta = 150^\circ + K\cdot 180^\circ$

$K=0 \rightarrow 30^\circ, 60^\circ, 120^\circ, 150^\circ$

$K=1 \rightarrow 210^\circ, 240^\circ, 300^\circ, 330^\circ$

$K=2 \rightarrow$ Answers not in $[0^\circ, 360^\circ)$

Nov 4-10:37 AM

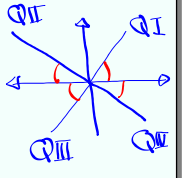
Solve $\tan^2 \frac{1}{2}x - 1 = 0$ in radians.

$\tan^2 \frac{1}{2}x = 1$

$\tan \frac{1}{2}x = \pm\sqrt{1}$

$\tan \frac{1}{2}x = \pm 1$

R.A. $\frac{\pi}{4}$



QI: $\frac{1}{2}x = \frac{\pi}{4} + K\cdot\pi$

$x = \frac{\pi}{2} + K\cdot 2\pi$

QII: $\frac{1}{2}x = \pi - \frac{\pi}{4} + K\cdot\pi$

$x = 2\pi - \frac{\pi}{2} + K\cdot 2\pi$

$x = \frac{3\pi}{2} + K\cdot 2\pi$

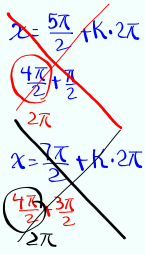
QIII: $\frac{1}{2}x = \pi + \frac{\pi}{4} + K\cdot\pi$

$x = 2\pi + \frac{\pi}{2} + K\cdot 2\pi$

QIV: $\frac{1}{2}x = 2\pi - \frac{\pi}{4} + K\cdot\pi$

$x = 4\pi - \frac{\pi}{2} + K\cdot 2\pi$

$x = \frac{7\pi}{2} + K\cdot 2\pi$



$K=0 \rightarrow \frac{\pi}{2}, \frac{3\pi}{2}$

$K=1 \rightarrow$ outside of $[0, 2\pi)$

Nov 4-10:45 AM

Solve $2\cos^2 x + \cos x = 0$ over $[0°, 360°)$

Hint: Factor LHS, Use Zero-Factor Property

Zero-Product Rule

If $A \cdot B = 0$, then $A = 0$ or $B = 0$ (Maybe both)

$\cos x \left[2\cos x + 1 \right] = 0$

$\cos x = 0$

$x = 90°, 270°$

$2\cos x + 1 = 0$

$\cos x = -\frac{1}{2}$ ← Ref. Angle $60°$

QII $\rightarrow x = 180° - 60° = 120°$

QIII $\rightarrow x = 180° + 60° = 240°$

$\{90°, 120°, 240°, 270°\}$

Nov 4-10:55 AM

Solve $\tan x \sin x + \sqrt{3} \sin x = 0$ on $[0, 2\pi)$.

$\sin x \left[\tan x + \sqrt{3} \right] = 0$

$\sin x = 0$

$0, \pi, 2\pi$

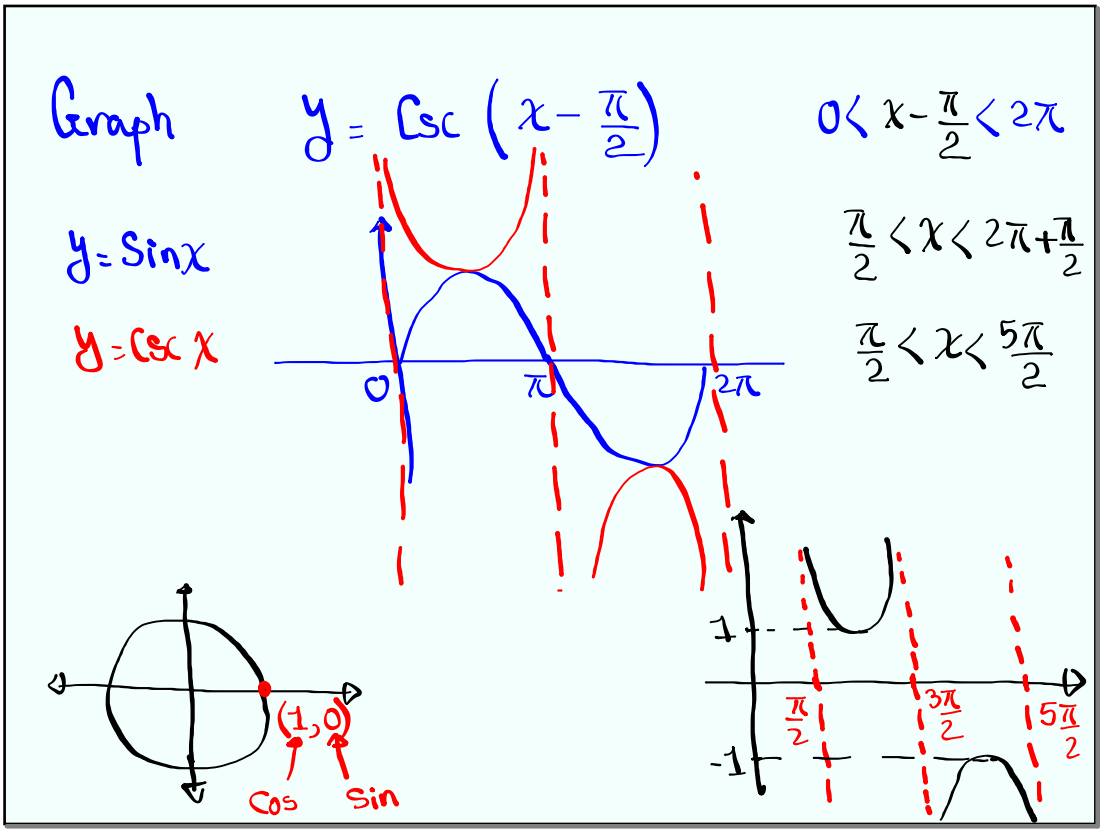
$\tan x = -\sqrt{3}$ ← RA. $\frac{\pi}{3}$

QII $x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

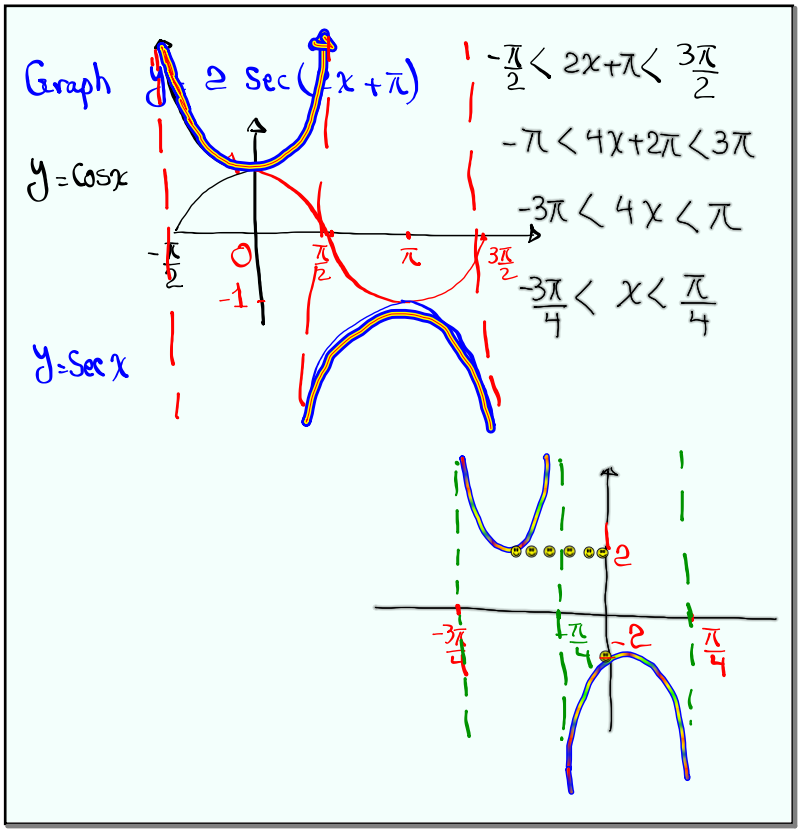
QIV $x = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$\left\{ 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{5\pi}{3} \right\}$

Nov 4-11:01 AM



Nov 4-11:09 AM

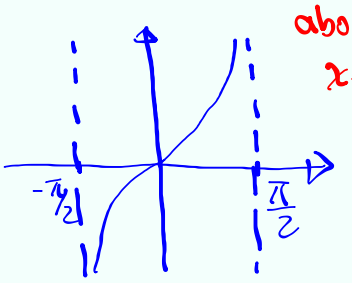


Nov 4-11:15 AM

Graph $y = -\tan\frac{1}{4}\pi x$

Reflection about x-axis

$y = \tan x$



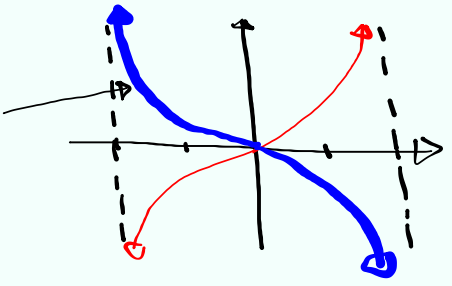
$-\frac{\pi}{2} < \frac{1}{4}\pi x < \frac{\pi}{2}$

$-2\pi < \pi x < 2\pi$

$-\frac{2\pi}{\pi} < x < \frac{2\pi}{\pi}$

$-2 < x < 2$

Final Graph



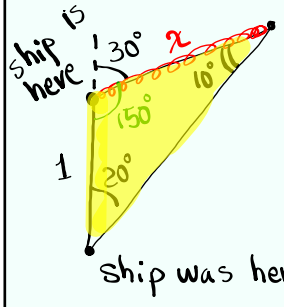
Nov 4-11:22 AM

A ship is sailing North.

It spots a lighthouse with a bearing of N20° E.

1 mile later, the bearing was N30° E.

How far is the ship from the lighthouse now.



$\frac{\sin 10^\circ}{1} = \frac{\sin 20^\circ}{x}$

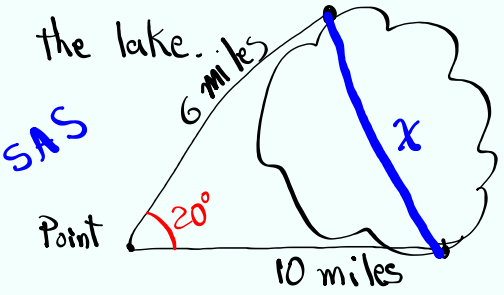
$x \cdot \sin 10^\circ = \sin 20^\circ$

$x = \frac{\sin 20^\circ}{\sin 10^\circ}$

$x = 1.9696\ldots \approx 2$ miles

Nov 4-11:26 AM

A point on one side of a lake is 6 miles & 10 miles from each end of the lake.



SAS

the angle between these two sights is 20°.

How wide is the lake

$x^2 = 10^2 + 6^2 - 2 \cdot 10 \cdot 6 \cdot \cos 20°$

$x^2 = 23.237$

$x \approx 5$ miles

Nov 4-11:36 AM